STATISTICAL ANALYSIS OF DATA SERIES IN TDLS

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Allan plots were proposed initially to analyze long-term laser frequency stability [1]. In TDLS it was used for the first time for concentration measurements in [2]. Allan approach in addition to FFT is efficient for analysis of any data series. In the paper we'll present experimental investigation and model calculations of Allan plots for main noises models: white noise, white noise after different filters, flicker noise, drift. Several examples of software developed for real time calculation of FFT and Allan plots are presented. Some applications in TDLS of software developed are considered.

1. D.W. Allan, Proc. IEEE 54, 221-230 (1966).
2. P.Werle, R.Mucke, F.Slemr, Appl. Phys. B 57, 131-139 (1993).

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Fourier Transform

$$Y(f) = \int_{-\infty}^{\infty} Y(t)\exp(-i2\pi ft)dt$$

Different calibrations are using for FT for particular application. For calibration presented above FT is unitary operator.

When stationary noise Y(t) is considered, its spectral density G is equal

$$G_Y(f) = \int_{-\infty}^{\infty} \exp(-j2\pi ft_1)dt_1 \int_{-\infty}^{\infty} \exp(j2\pi ft_2)dt_2 \langle Y(t_1)Y(t_2)\rangle$$

$$g_Y(f) = \sqrt{G_Y(f)}$$

Here <> is averaging over realizations

Spectral densities of shot and Johnson noises

Shot noise	Johnson noise
$G_i(f) = ei$	$G_U(f) = 2kTR$
i - current	T- temperature, R - resistance

Fast Fourier Transform

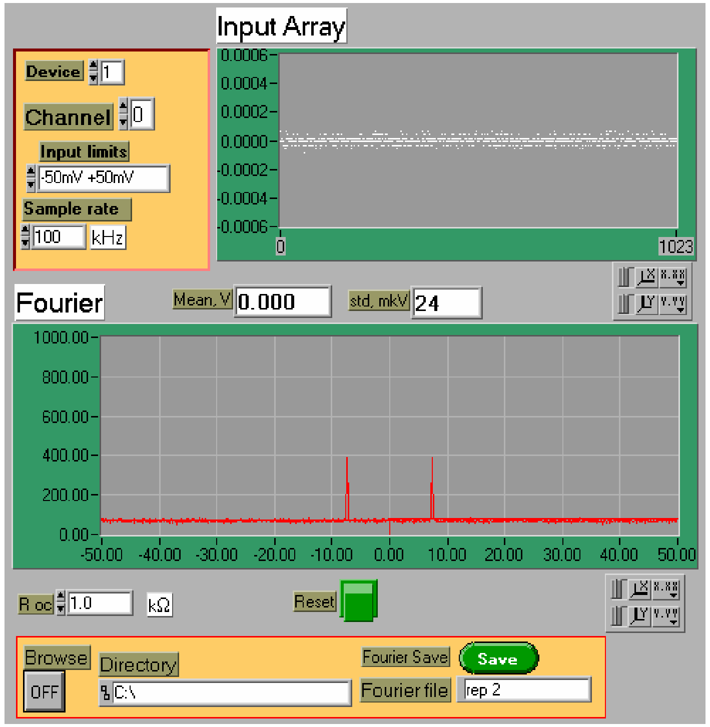
For discrete massive A(n) (n = 1,…N) Fast Fourier Transform (FFT) is using.

$$FFT_A(k) = \sum_{n=1}^{N} A(n) \exp\left[\frac{i2\pi}{N} kn\right]$$

When stationary noise Y(t) is considered, its spectral density G is equal

$$G_A(k) = \sum_{n=1}^{N} \exp\left[\frac{i2\pi}{N} kn\right] \sum_{m=1}^{N} \exp\left[-\frac{i2\pi}{N} km\right] \langle A(n)A(m) \rangle$$

Here <> is averaging over realizations



Example of software interface for real time noise spectral density measurements.

$$A(n) = A_0 \cos\left[\frac{2\pi i}{N} nK\right] \quad \Rightarrow \quad G_A(k) = \frac{A_0 N}{2} \sum_{p=-\infty}^{\infty} \delta_{K+pN,k}$$

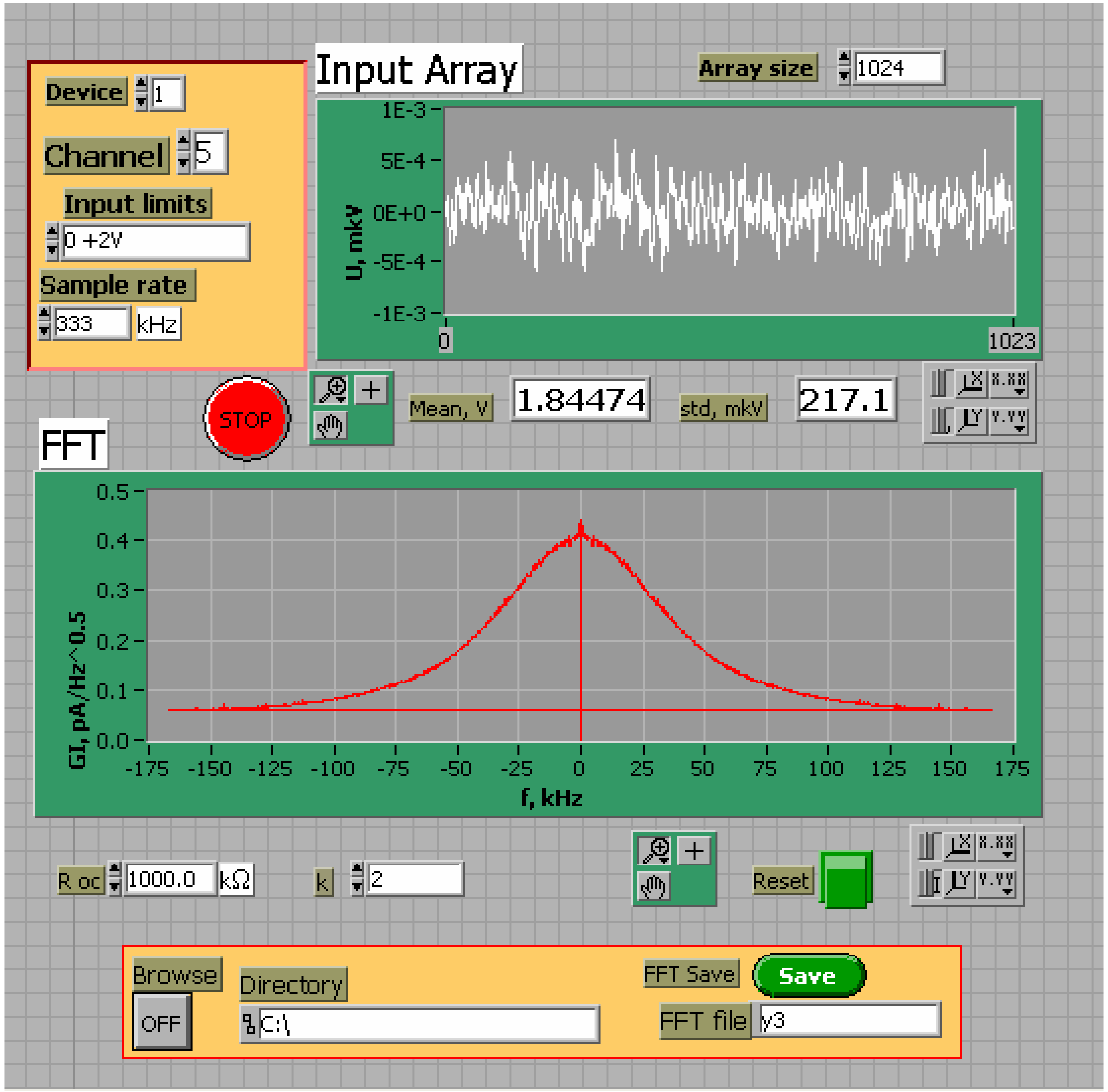
Identification of cross talking origin.

For white noise

$$\langle A(m)A(n) \rangle = A_0^{\ 2} \delta_{m,n} \Rightarrow G_A(k) = A_0 \sqrt{N}$$

FFT Software

Several software modifications were developed to measure FFT for different applications

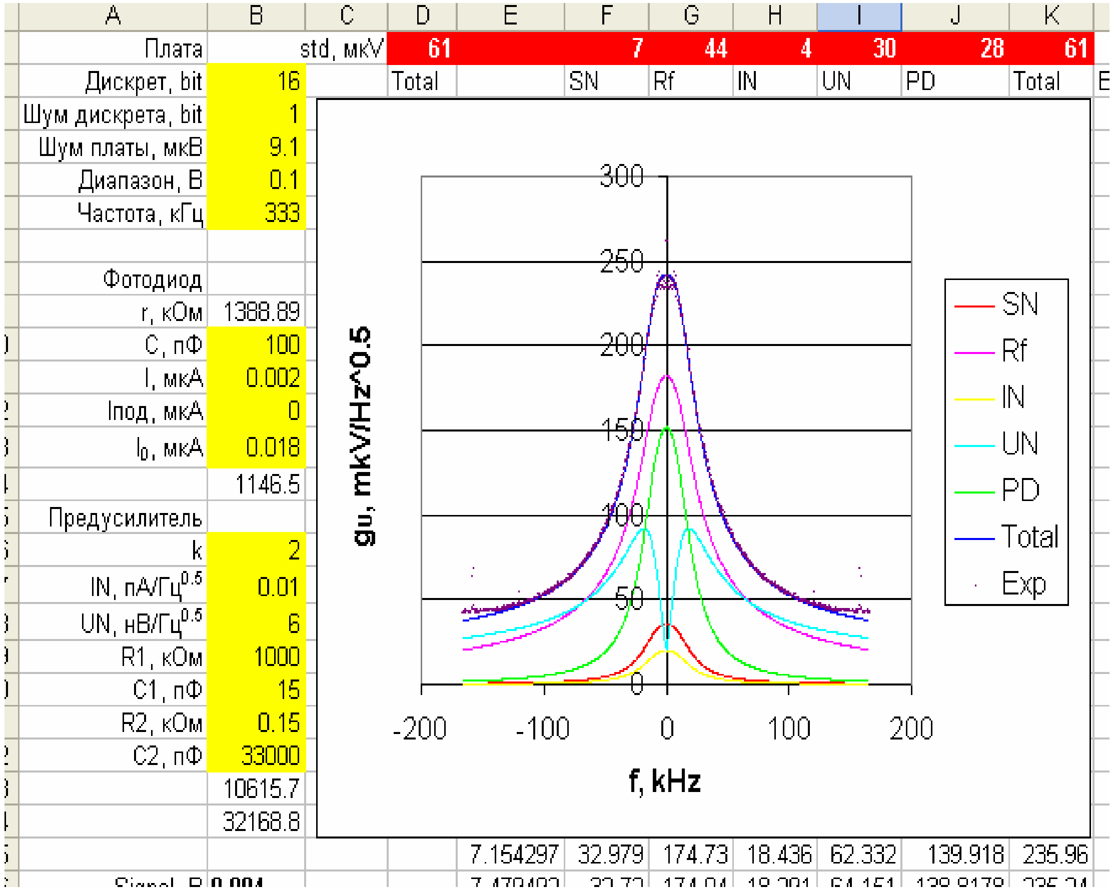


Interface of FFT PD software developed to measure photocurrent noise spectral density.

Example of software operation when photo-diode measured sky radiance.

In present case shot noise dominated. Spectrum shape is determined by amplifier.

Photo-Diode +Amplifier Noise



Each diode laser and particular application needs optimization of photodiode and preamplifier in use.

Example of photodiode and preamplifier noise spectral density measurements.

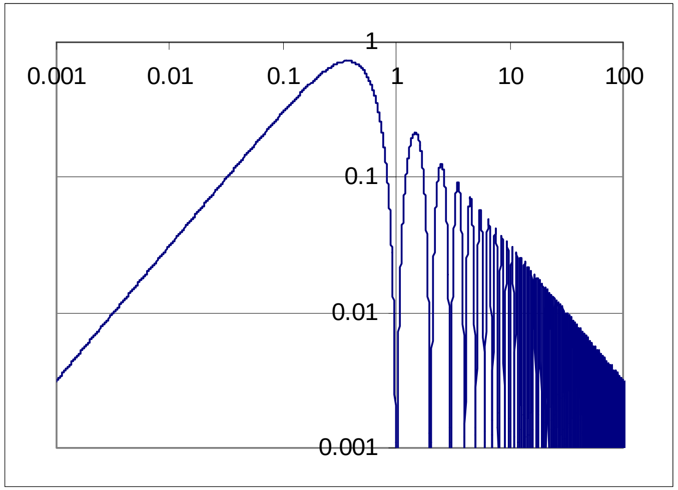
Different noise mechanisms are considered and compared with experimentally measured noise spectrum.

Allan Deviation

Allan plot is efficient for analysis of any data series and identification of different noise types.

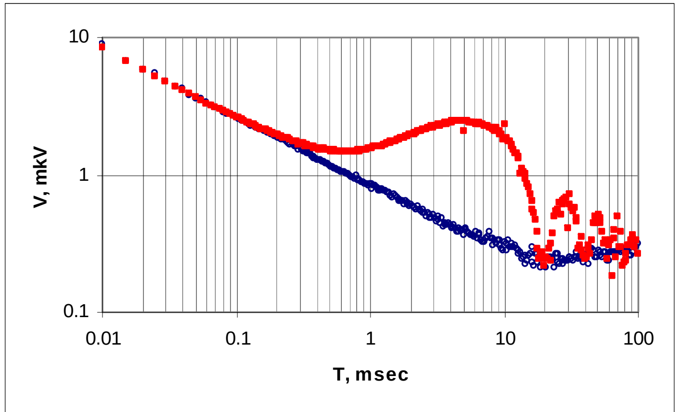
$$\sigma_A(K) = \mathrm{std}[S_K(n)], \qquad \text{где}$$

$$S_K(n) = \frac{1}{\sqrt{2K}}\left[\sum_{i=K(n+1)}^{K(n+2)} S(i) - \sum_{i=Kn}^{K(n+1)} S(i)\right]$$



Allan plot for harmonic signal V=C cos(2πft)

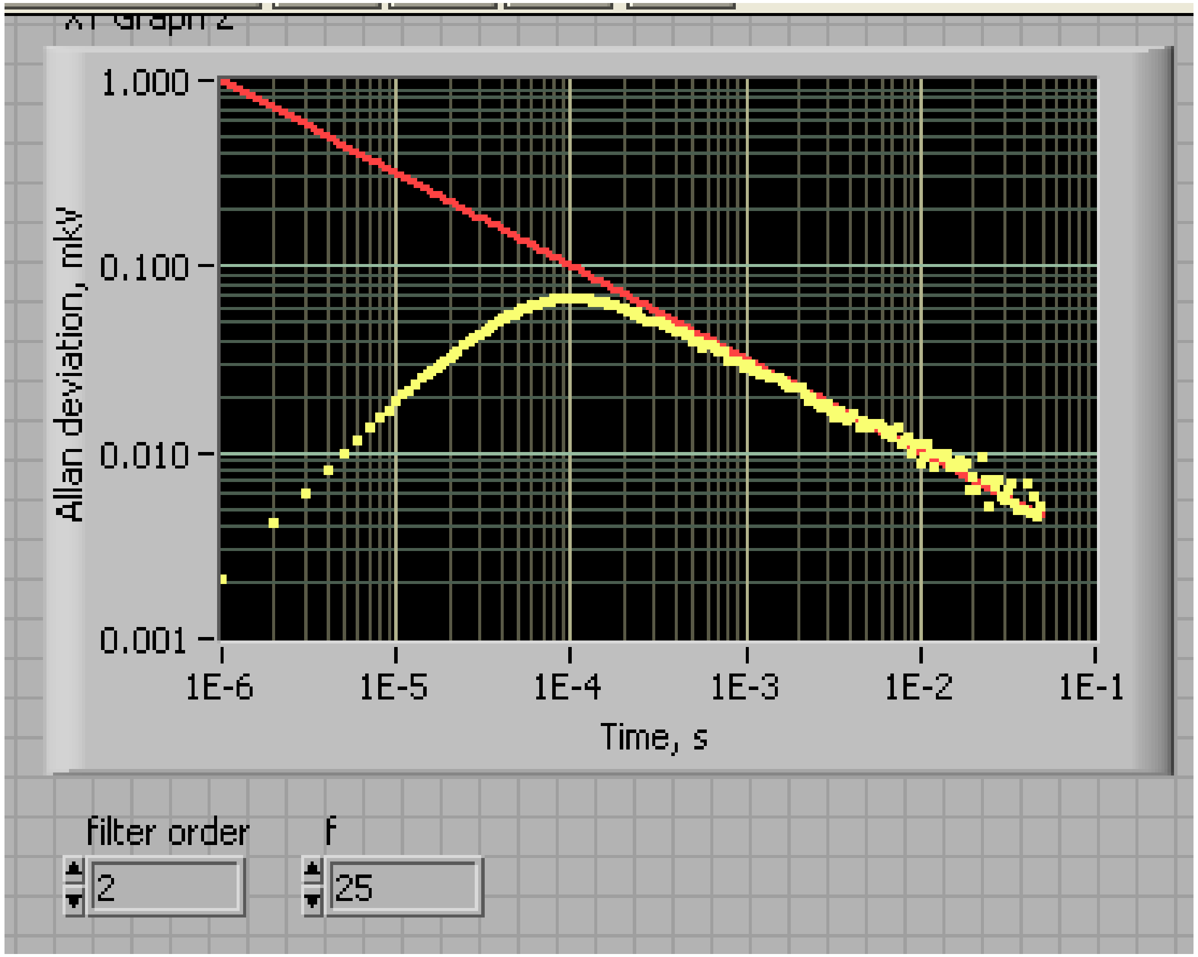
$$\sigma_A = C\frac{|1-\cos(2\pi f T)|}{2\pi f T}$$



Allan plot of signal containing small 50 Hz cross talking

White Noise

Special software was developed to model noise investigation and compare in with theoretical calculations



Example of software operation.

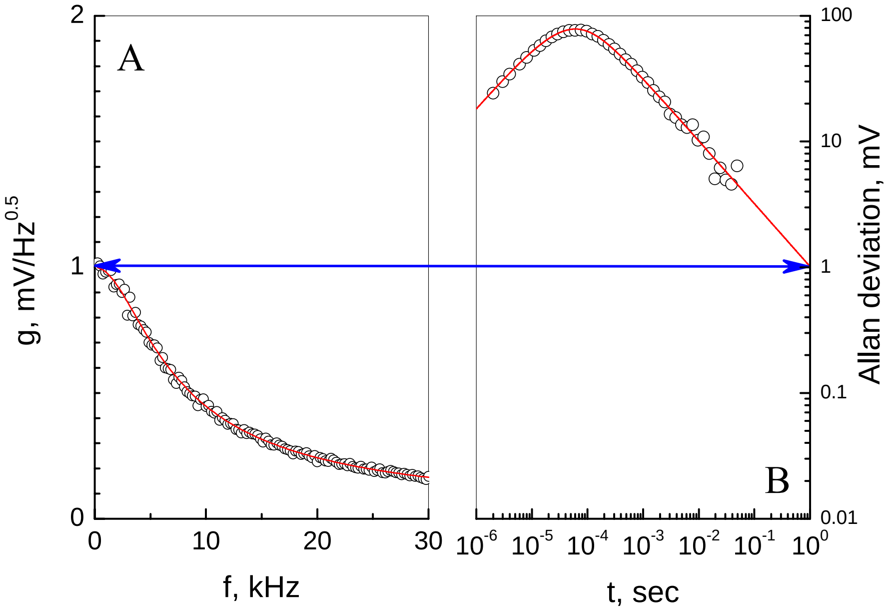
Computer generated Gaussian white noise with std =1.

Generated noise passed Bessel filter (2 order filter in present case).

Allan plot of initial (red line) and filtered noise (points).

White Noise

Noise spectral density (left) and Allan plot (right) for white noise after first order filter, open cycles – experiment, red lines – calculation. White noise after 1st order Bessel filter.



$$\langle U(t)U(t+\tau)\rangle = \sigma_0{}^2 \exp(-2\pi B|\tau|) \qquad \sigma_0{}^2 = \pi G_0 B$$

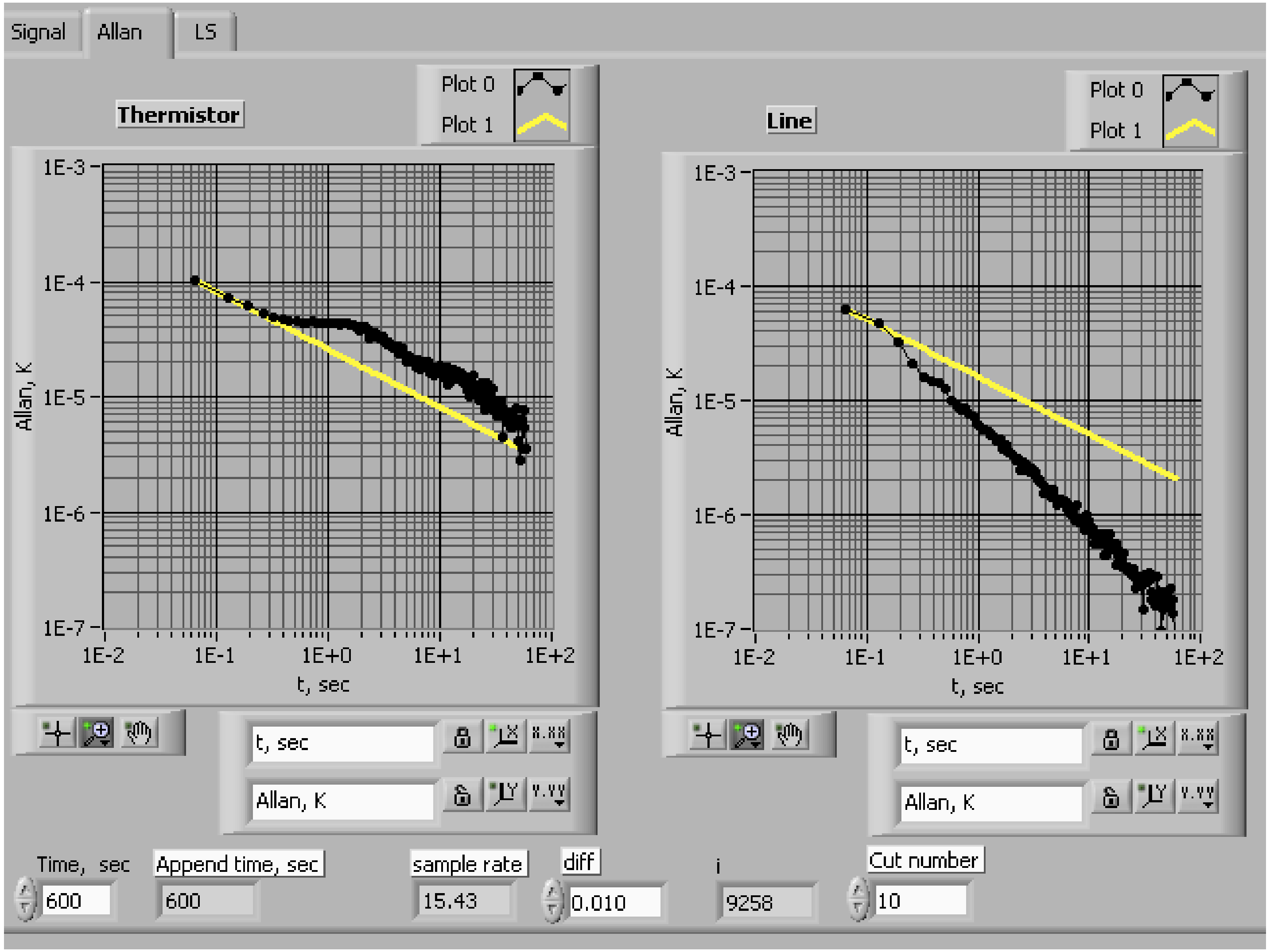
$$\sigma_A(K) = \sigma_0 \sqrt{\frac{(1-e^{-CK})}{K^2}\left\{K + 2\frac{K-1}{e^C - 1} - 2\frac{e^{-C} - e^{-CK}}{(e^C - 1)(1-e^{-C})}\right\}}$$

$$C = 2\pi B \Delta t$$

Important: values of noise spectral density at f = 0 and Allan deviation at t = 1 sec have to be the same. This result does not depend on filter type. Hence, it is good check of experimental accuracy. Moreover, either spectral density or parameters of Allan plot can be determined from independent measurements.

It necessary to mention, that many references failed with this fundamental check.

Allan plots of different signals



Special software was developed for simultaneous measurement and analyses of signals in any two channels.

Allan plot of temperature as measured by termistor (left) and using spectral line (right). Both temperature stabilization and frequency tuning cycles stabilization are on.

DL frequency stability

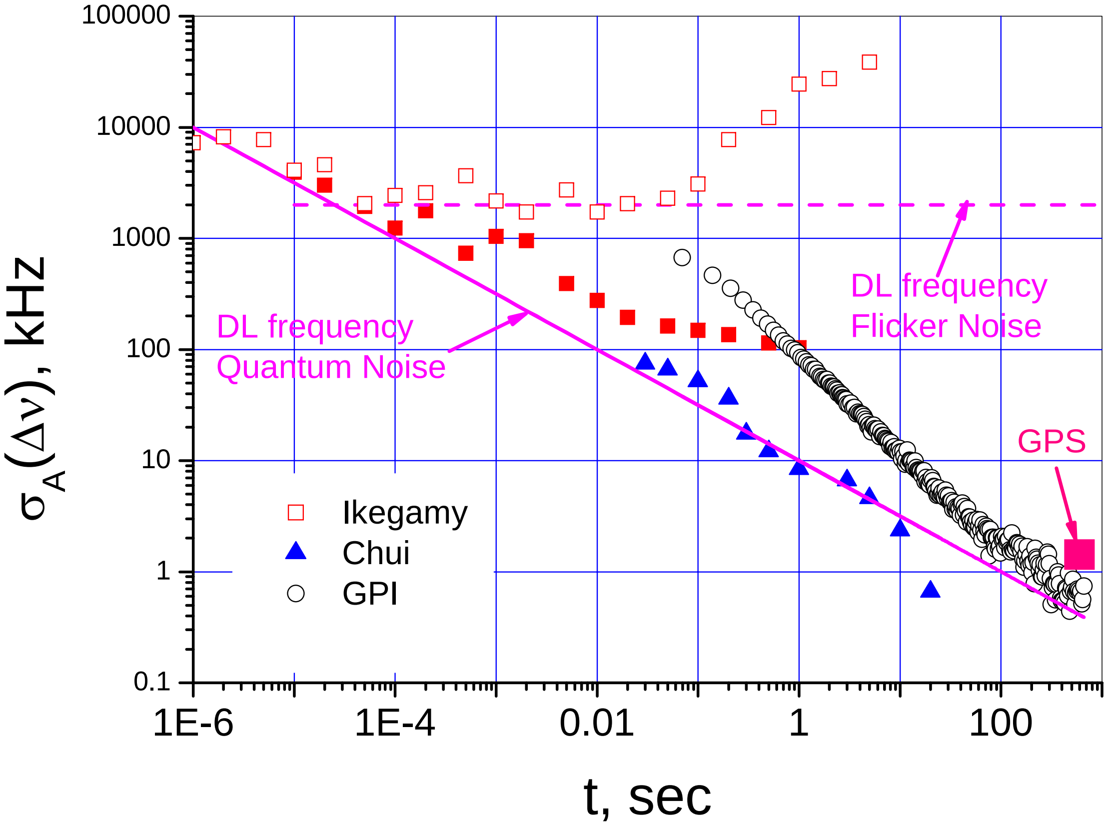
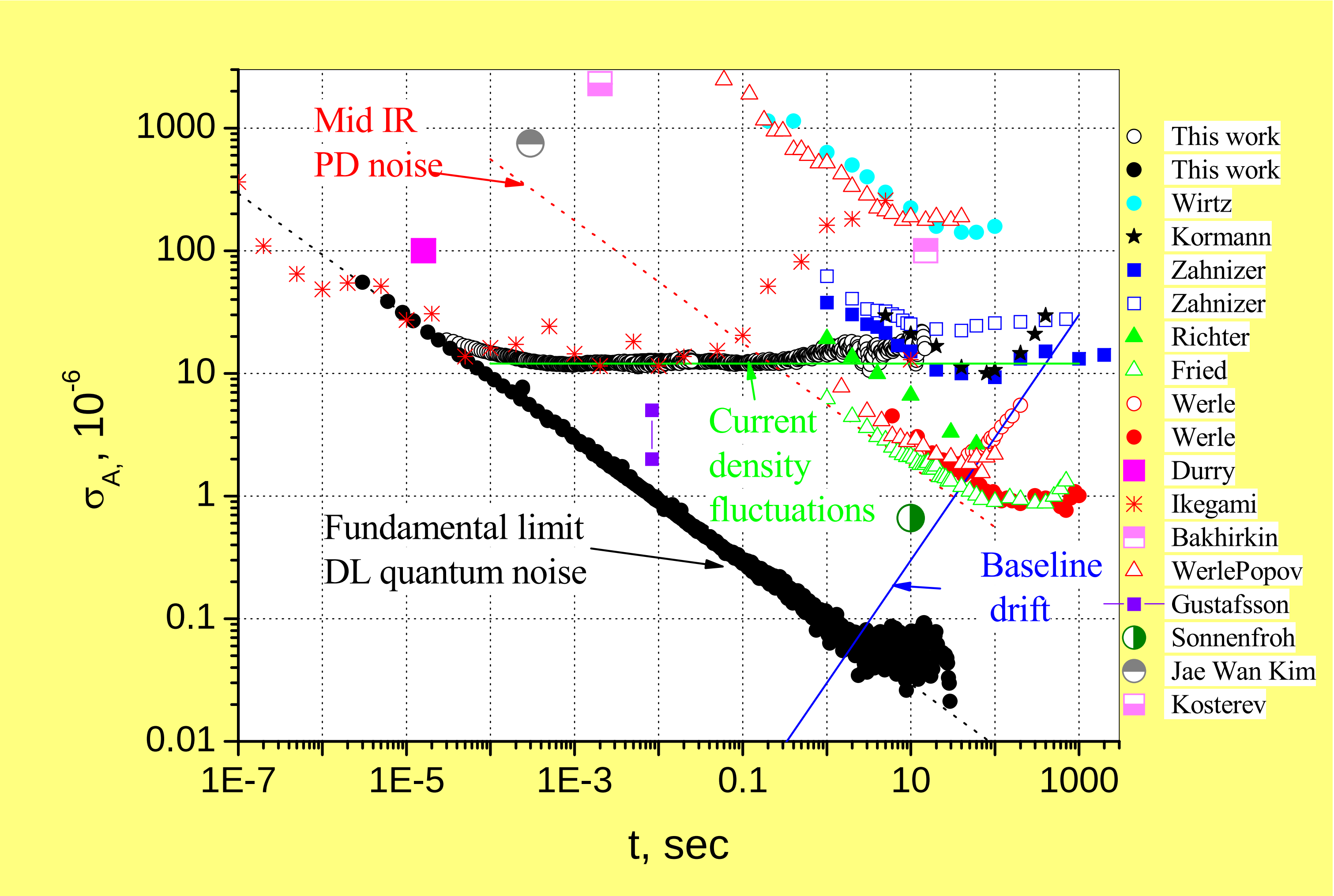


Fig. shows Allan deviation of DL frequency demonstrating kHz stability close to needs of Global Position System (GPS). In present case spectral line of water vapor at low pressure (WHH ~ 600 MHz) was used for stabilization.

Comparison with experiments of traditional frequency stabilization are presented. Dominating noise mechanisms such as DL frequency quantum noise and flicker noise due to excitation current density fluctuations are considered.

Sensitivity limits of TDLS



As example Allan plot of absorption sensitivity (minimum detectable absorption, noise equivalent absorption) of the best known to author results is presented. Main physical mechanisms limitting sensitivity are shown.